PRODUCT CHOICE AND PRODUCT SWITCHING

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Abstract

This paper develops a model of endogenous product selection within industries by firms. The model is motivated by new evidence we present on the prevalence and importance of product-changing activity by U.S. manufacturers. Three-fifths of continuing firms alter their product mix within an industry every five years, and added and dropped products account for a substantial portion of firm output. In the model, firms make decisions about both industry entry and product choice. Product choice is shaped by the interaction of heterogeneous firm characteristics and diverse product attributes. Changes in market conditions within an industry result in simultaneous adjustment along a number of margins, including both entry/exit and product choice.

Keywords: Product selection, heterogeneous firms, product differentiation, sunk entry costs

JEL classification: L11, D21, L60

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1. Introduction

Product selection and product turnover are fundamental dimensions of firm activity, but receive less attention than many other aspects of industrial organization. Most theoretical models of firm behavior take product choice as given or treat the decision to enter a product market to be the same as the decision to create the firm. Similarly, there has been no systematic empirical examination of product-mix changes by firms over time and across industries.

We introduce a set of stylized facts about firm product selection that is generated from a comprehensive sample of U.S. manufacturing firms between 1972 and 1997. These facts document the surprising prevalence and significance of firm product-mix changes within industries over time. In any five year interval, 60 percent of surviving manufacturing firms add or delete products within industries where they currently produce.[1] Within firms, recently added and dropped products account for an average of 37 percent and 47 percent of firm output, respectively. Across all firms producing multiple products, recently added and dropped products make up a third of total manufacturing output.

We capture the essence of these findings in a theoretical model that combines heterogeneous firms, heterogeneous products, and steady-state market entry and exit in general equilibrium. Firms choose endogenously whether to enter or exit an industry, as well as which product within the industry to produce. Firms have varying productivity levels and products are heterogenous both in terms of how they are demanded by consumers and in their production technology. To enter the industry, firms pay a sunk cost of entry, subsequently learn their productivity, and then decide whether to begin producing or to exit. If they decide to enter, they then choose which product to produce. The general equilibrium of the model characterizes firms' entry/exit decisions, the choice of which product to make, the mass of firms, output and consumption of the two products, as well as real income and welfare.

Some of the features of our model, such as heterogeneous firms, ongoing entry and exit, and positive covariation in entry and exit rates across industries due to underlying variation in sunk entry costs, are present in existing theoretical and empirical studies of industry equilibrium. Other elements, specifically endogenous product selection and the ability of firms to change their product mix, are new.

[1] Throughout this paper we consider products changes within the existing set of industries produced by the firm. A firm 'adds' a product when it starts producing a new product within its existing industry mix. See Section 2 for the precise definition of 'product' and 'industry'.

In addition to accounting for many of the new stylized facts about firm product choice, the model yields three main results. First, firm product choice is shaped by the interaction of firm characteristics, product characteristics and market conditions in the industry as a whole. Firms endogenously sort into products based on both product attributes and firm characteristics. In our case, products are distinguished by variation in fixed and variable costs of production and by substitutability in demand, while firms vary in terms of their productivity. In equilibrium, high productivity firms self-select into the high fixed cost product regardless of the relative size of the variable costs.

Second, a firm's entry and exit decision is systematically related to the decision of which product to manufacture. With changes in market conditions within the industry, such as sunk costs of entry or demand, firms reconsider both their decision to produce and their decision about which product to make. The analysis emphasizes a variety of adjustment margins through which an industry responds to exogenous changes in the economic environment: product choice, entry and exit, the mass of firms, average firm size, and average firm productivity. In general, adjustment will occur simultaneously along all of these margins.

Third, our theoretical analysis, as well as our stylized facts, challenges the way in which firms and industries are traditionally viewed. Much economic research portrays firms as highly specialized entities producing the same differentiated product from birth to death. Our analysis emphasizes that firms frequently change what they do and that these product changes account for a sizeable share of firm output. Furthermore, product switching has important implications for economic outcomes at both the firm and industry level. If products have different production technologies, switches between products will be responsible for changes in measured production technique at the firm-level. Similarly, changes in composition of products across firms will give rise to changes in measured production technique at the industry-level.

Several implications of the theoretical model are amenable to empirical analysis and the final section of the paper takes them back to the data. We first examine the differences in products within industries. Firms making distinct products have statistically significantly different productivities, consistent with the systematic sorting across products by heterogeneous firms in the model. Second, we ask whether industries with greater changes in firm birth and death rates (new and dying firms) are also characterized by more product switching by continuing firms. We confirm the model's predictions of simultaneous adjustment along a number of margins by showing that product entry and exit (by continuing firms) and firm entry and exit are highly correlated within industries.

Our analysis relates to existing empirical work, which has documented firm heterogeneity and the importance of entry and exit in understanding industry dynamics, including Aw, Chung, and Roberts (2003), Bartelsman and Doms (2000), Bernard and Jensen (1995), Davis and Haltiwanger (1991), Dunne, Roberts, and Samuelson (1988, 1989), Olley and Pakes (1996) and Pavcnik (2002). It is also linked to theoretical research which has sought to formalize these features of the world, including in particular Melitz (2003), and also Bernard *et al.* (2003), Hopenhayn (1992), Jovanovic (1982), and Yeaple (2002). Our finding that firm birth and death rates are correlated with product additions and drops suggests that the U.S. manufacturing sector is even more dynamic than the recent work on entry and exit has suggested.

Finally, our research relates to broader work in industrial organization literature on firms and products. Closest to our concerns is the work of Sutton (1998, 2002) on firm capability and its implications for the product trajectories of firms. Classic treatments on product choice include Hotelling (1929), Chamberlain (1951) and Lancaster (1966). More formal treatments of horizontal and vertical differentiation include Dixit and Stiglitz (1977), Shaked and Sutton (1982, 1987), and Spence (1976).[2]

The key contributions of this paper are to present empirical evidence on product choice as an important and neglected margin of adjustment by firms, to develop a theoretical model of endogenous product selection by heterogeneous firms in industry equilibrium, and to trace the implications of endogenous product choice for firms, industries and economies.

The remainder of the paper is structured as follows. Section 2 provides evidence on the importance of product changes by U.S. manufacturing firms from 1972 to 1997. Section 3 develops the theoretical model and Section 4 solves for industry equilibrium. Section 5 derives the main theoretical results on product choice as a margin of adjustment by firms. Section 6 examines the empirical evidence on these results. Section 7 concludes. An appendix at the end of the paper collects together proofs and technical derivations.

[2] In our theoretical analysis, we focus on some of the most important dimensions of firm product choice: heterogeneous firms, diverse products, and market conditions in the industry as a whole. In reality, many other considerations will play a role, including corporate diversification, mergers and divestitures, as well as evolving firm capabilities and factor accumulation. See, among others, Amihud and Baruch (1981), Bolton and Farrell (1990), Chandler (1990), Helfat and Raubitschek (2000), Milgrom and Roberts (1990), and Montgomery (1994).

2. Product Changes By U.S. Manufacturing Firms, 1972 to 1997

This section documents product-mix changes by U.S. manufacturing firms across five-year intervals from 1972 to 1997.[3] As noted above, we focus on within-industry changes to a firm's product mix, i.e. additions of products within existing industries. We demonstrate that a majority of firms engage in this activity and that added and dropped products represent a substantial fraction of firm output.

The data are derived from the U.S. Censuses of Manufactures of the Longitudinal Research Database (LRD) managed by the U.S. Census Bureau. Manufacturing Censuses are conducted every five years, and we examine data from 1972 to 1997. The sampling unit for each Census is a manufacturing establishment, or plant, and the sampling frame in each Census year includes information on plants' product-level output. Because product-mix decisions are made at the level of the firm, we aggregate this information up to the firm level for all of the results reported below.[4] This aggregation means that we do not examine intra-firm, inter-plant product shuffling. Our results on the extent and importance of product switching are based on an average of 141,561 surviving firms in each Census year. Roughly one-third of manufacturing firms exit between Census years and one-third of firms are new in any given Census. Our focus on firms that survive from one Census to the next nets out the product-changing activities of exiting and entering firms, i.e. we do not record an exiting firm as a firm that drops all of its existing products. However, our theoretical model allows for endogenous firm exit as well product selection and when we examine empirical implications of the model, we consider firm entry and exit.

Our definitions of "industry" and "product" are based upon 1987 Standard Industrial Classification (SIC) categories which group manufacturing products according to their underlying production attributes.[5] We refer to four-digit SIC (SIC4) categories as industries and five-digit SIC (SIC5) categories as products.[6] These groupings motivate our theoretical

[3] Existing empirical research on the product mix of U.S. manufacturing firms has focused on product diversification. Gollop and Monahan (1991), for example, show that firm product-mix diversification increased in most two-digit SIC industries between 1963 and 1982.

[4] Product-mix data is not available for some small manufacturing plants (so-called Administrative records). We exclude any firm-year observation where any plant within the firm does not record product-level data.

[5] Use of a classification system like the SIC to identify firm output limits our ability to detect innovation in two ways. First, though we can observe product additions that expand a firms range of products within an industry, we have no information about new products added within five-digit SIC categories. Second, because the SIC classification is changed infrequently, we cannot distinguish between additions that are new to the market verus new to the firm.

[6] Our terminology differs slightly from that of the U.S. Census Bureau: whereas we refer to SIC5 categories

modelling of product choice by firms within industries. Five-digit categories are relatively aggregate and switching between them typically represents a substantive business decision by the firm.

Table 1: Five-Digit SIC Products in Four-Digit SIC Industry 3357 (Nonferrous Wiredrawing and Insulating)

SIC		Description
3357		Nonferrous Wiredrawing and Insulating
33571	Products	Aluminum Wire
33572		Copper Wire
33573		Other Nonferrous Metal Wire
33575		Nonferrous Wire Cloth
33576		Apparatus Wire and Cord Sets
33577		Magnet Wire
33578		Power Wire
3357A		Electronic Wire
3357B		Telephone Wire
3357C		Control Wire
3357D		Building Wire
3357E		Other Wire NES
33579		Fiber Optic Cable

Source: U.S. Census Bureau (1996).

Table 1 provides a sense of the relative level of detail between products and industries under the SIC by listing the thirteen SIC5 products under the SIC4 industry Nonferrous Wiredrawing and Insulating (SIC 3357). The products in this industry range from copper wire (33571) to fiber optic cable (33579). The products differ in terms of both end use and in terms of the inputs and technologies required to manufacture them. These differences across products provide the motivation for our subsequent theoretical assumptions of imperfect substitutability and variation in production technique.

For each firm that survives from one Census year (t) to the next ($t+5$), we record the set of products and industries produced in each year. Using these data, we are able to

as products, the Census Bureau refers to them as "product classes". In Census years, plant output is recorded according to either five-digit or seven-digit SIC categories. Roughly 7,000 of the 15,000 seven-digit categories are recorded directly in the LRD; the rest are recorded according to their more aggregate SIC5 category. As a result, to obtain a complete and consistent set of products for all manufacturing firms we aggregate any seven-digit categories in the sample up to the roughly 1,500 five-digit SIC products that are available for all firms. For a complete list of products, see U.S. Census (1996) available at http://www.census.gov/prod/2/manmin/mc92-r-1.pdf.

examine the importance of within-industry changes in product mix. 'Dropped' products (SIC5) are present in the year t product mix but absent from the year $t+5$ product mix. For a product (SIC5) to be 'Added', it must be absent from the year t product mix, present in the year $t+5$ product mix, and lie in the set of industries (SIC4) produced in year t.[7]

Table 2: Product Changes by U.S. Manufacturing Firms, 1972 to 1997

Product-Changing Activity	Percent of Firms
Firm takes no action (within industry)	40
Firm drops products only	19
Firm adds products only	7
Firm both adds and drops products	35

Note: Table displays average share of surviving U.S. manufacturing firms engaging in each type of product-changing activity across five-year intervals from 1972 to 1997. Products refer to five-digit SIC categories and product changes refer to the addition or deletion of products from the set of currently produced four-digit SIC industries. There are an average of 140,000 surviving firms in each Census year.

The extent of firm product-changing activity across five-year periods between 1972 and 1997 is summarized in Table 2. This table separates firms into four mutually exclusive categories based on their product-changing activities: (1) firms that neither add nor drop products from the set of currently produced four-digit industries between years t and $t+5$; (2) firms that drop at least one product but do not add a product[8]; (3) firms that add at least one product but do not drop any products; and (4) firm that both drop at least one product and add at least one product within the set of currently produced SIC4 industries.

The results in Table 2 clearly document the pervasive nature of product-switching by U.S. manufacturing firms. An average of 60 percent of continuing firms alter their product mix in the five years between Censuses by either dropping a product or adding a product. Simultaneous changes in both directions, adding and dropping at least one product, is the most prevalent product-changing activity, occurring at an average of 35 percent of surviving

[7] For the remainder of the paper, all discussion of products that are added or dropped will refer to these within-industry definitions.

[8] A firm may add a product outside its year t industry mix and still be in this category.

firms (58 percent of firms that make any change to their product mix). Dropping without adding, and adding without dropping, are less frequent events, occurring in 19 percent and 7 percent of firms, respectively. In formulating the theoretical model in the next section, we focus on the most common form of within-industry product changes, i.e. simultaneous additions and deletions of products.

Table 3: Output Share of Added and Dropped Products, 1972 to 1997

	Added-Product Share	Dropped-Product Share
Firms that Add	37	-
Firms that Drop	-	47

Notes: Table reports the mean share of multiple-product firm output represented by added and dropped products between 1972 and 1997. Added-product share is the share of firm year t+5 output due to products added to firms' existing sets of industries between years t and t+5. Dropped-product share is the share of year t output represented by products dropped between years t and t+5. Products and industries refer to five-digit SIC and four-digit SIC categories, respectively.

One possible concern about the prevalence of product-switching is that it might account for a small fraction of activity at the firm, i.e. that these are minor products. To gauge the importance of product switching for the firm itself, we calculate the fraction of output embodied in added and dropped products in Table 3.

The products that firms add and drop represent a substantial portion of firms' overall output. Table 3 reports that products added between Census years t and $t+5$ account for an average of 37 percent of adding firms' output in year $t+5$, while products dropped between years t and $t+5$ comprise an average of 47 percent of dropping firms' year t output. We also find that across all multiple-product firms, recently added and dropped products make up a third of total manufacturing output.

This section has provided the first set of facts on product-switching across the whole of U.S. manufacturing. Changes in product mix are widespread across firms and represent significant adjustments to output by firms. In the next section, we construct a model of firm product selection that is motivated by these findings. In constructing the model, we are careful to incorporate previously known facts about firms and industries - e.g., firm heterogeneity and the importance of entry/exit - while also explaining the new dimensions of the data uncovered here.

3. A Heterogeneous Firm Model of Industry Entry and Product Choice

Consider an economy consisting of a single industry within which consumers and firms decide whether or not to consume and produce a number of distinct products. A key concern of the analysis will be the relationship between firms' decision whether or not to enter an industry and their decision of which product within the industry to produce. To keep the analysis as simple as possible, we consider the case of a single industry where heterogeneous firms choose whether to produce one of two products.[9] Within the industry, consumers have a taste for both products, as represented by the following CES utility function:

$$U = [aC_1^{\nu} + (1-a)C_2^{\nu}]^{1/\nu}. \tag{1}$$

where a captures the relative strength of preferences for the two products, and we assume that the products are imperfect substitutes with elasticity of substitution $\psi = \frac{1}{1-\nu} \in (1,\infty)$. Firms produce horizontally differentiated varieties of the products, captured here by C_i which is a consumption index defined over firm varieties ω in each product market i:

$$C_i = \left[\int_{\omega\in\Omega_i} q_i(\omega)^{\rho} d\omega\right]^{1/\rho}, \qquad P_i = \left[\int_{\omega\in\Omega_i} p_i(\omega)^{1-\sigma} d\omega\right]^{1/1-\sigma}. \tag{2}$$

where $\{\Omega_i\}$ is the set of available varieties in market i, P_i is the price index dual to C_i, and $\sigma = \frac{1}{1-\rho} \in (1,\infty)$ is the elasticity of substitution between varieties of the same product, which is assumed to be greater than the elasticity of substitution between products: $\sigma > \psi$.

Consumer expenditure minimization yields the following expression for equilibrium expenditure (equals revenue, $r_i(\omega)$) on a variety:

$$r_i(\omega) = R_i\left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} = \alpha_i(\mathcal{P})\, R\left(\frac{p_i(\omega)}{P_i}\right)^{1-\sigma} \tag{3}$$

which is increasing in aggregate expenditure (equals aggregate revenue $R = R_1 + R_2 = \int_{\omega\in\Omega_1} r_1(\omega)d\omega + \int_{\omega\in\Omega_2} r_2(\omega)d\omega$), increasing in the share of expenditure allocated to product i, $\alpha_i(P_2/P_1) = \alpha_i(\mathcal{P})$, decreasing in own variety price, $p_i(\omega)$, and increasing in the price of competing varieties as summarized in the price index, P_i.

With CES utility, the share of expenditure allocated to product 1 is increasing in the relative price of product 2, $\mathcal{P} = P_2/P_1$ (since $\psi > 1$), and increasing in the relative weight

[9] It is straightforward to extend this framework to a multi-industry model and/or to allow firms to produce any of a finite number of products within an industry. As will become clearer below, these extensions merely complicate the analysis without changing the key insights of our approach.

given to product 1 in consumer utility, a:

$$\alpha_1(\mathcal{P}) = \left[1 + \left(\frac{1-a}{a}\right)^{\psi} \mathcal{P}^{1-\psi}\right]^{-1}, \qquad \alpha_2(\mathcal{P}) = 1 - \alpha_1(\mathcal{P}). \tag{4}$$

3.1. Production

As well as entering demand as imperfect substitutes, the products also have different production technologies. We consider the case where this difference in production technology takes the form of a difference in the fixed and variable costs of production. We assume that product 2 has a higher fixed cost of production: $f_2 > f_1$. Variable costs are indexed by the parameter b_i and, without loss of generality, we normalize $b_1 = 1$ and $b_2 = b$. We allow variable costs of production for product 2 to be either smaller or greater than those for product 1.

Labor is the sole factor of production and is supplied inelastically at its aggregate level L, which also indexes the size of the economy. The production technology follows Melitz (2003) in that variable cost is assumed to depend on heterogeneous firm productivity. We differ in that we allow for multiple products and hence endogenous product choice within the industry. The labor required to produce q_i units of a variety in product market i is given by:

$$l_i = f_i + \frac{b_i q_i}{\varphi} \tag{5}$$

so that the variable cost of production depends on b_i which is common to all firms as well as on the firm-specific productivity, φ.[10]

The existence of fixed production costs implies that, in equilibrium, each firm will choose to produce a unique variety. Profit maximization yields the standard result that equilibrium prices are a constant mark-up over marginal cost, with the size of the mark-up depending on the elasticity of substitution between varieties:

$$p_i(\varphi) = \left(\frac{\sigma}{\sigma - 1}\right) \frac{w b_i}{\varphi}. \tag{6}$$

We choose the wage as the numeraire so that $w = 1$. Using this choice of numeraire and the pricing rule in the expression for revenue above, equilibrium firm revenue and profits

[10] The assumption that fixed costs of production are independent of productivity captures the idea that many fixed costs, such as building and equipping a factory with machinery, are unlikely to vary substantially with firm productivity. All the analysis requires is that fixed costs are less sensitive to productivity than variable costs.

are:

$$r_i(\varphi) = \alpha_i(\mathcal{P})R\left(P_i\rho\frac{\varphi}{b_i}\right)^{\sigma-1} \tag{7}$$
$$\pi_i(\varphi) = \frac{r_i(\varphi)}{\sigma} - f_i.$$

One property of equilibrium revenue that will prove useful below is that the relative revenue of two firms with different productivity levels in the same product market depends solely on their relative productivity: $r_i(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_i(\varphi')$. Similarly, the relative revenue of two firms with different productivity levels in different product markets depends on their relative productivities, the relative variable cost of making the two products, the relative expenditure share devoted to the two products, and relative price indices:

$$r_2(\varphi'') = \left(\frac{1-\alpha_1(\mathcal{P})}{\alpha_1(\mathcal{P})}\right)\left[\left(\frac{\varphi''}{\varphi'}\right)\mathcal{P}\frac{1}{b}\right]^{\sigma-1} r_1(\varphi'). \tag{8}$$

3.2. Industry Entry and Exit

To enter the industry (and produce either product), firms must pay a fixed entry cost, $f_e > 0$, which is thereafter sunk. After paying the sunk cost, firms draw their productivity, φ, from a distribution, $g(\varphi)$. This formulation captures the idea that there are sunk costs of entering an industry and that, once these costs are incurred, some uncertainty regarding the nature of production and firm profitability is realized. Firm productivity is assumed to remain fixed thereafter, and firms face a constant exogenous probability of death, δ, which we interpret as due to *force majeure* events beyond managers' control.[11]

After entry, firms decide whether to begin producing in the industry or exit. If they decide to produce, they choose what product to make. For simplicity, we assume that firms make only one product or that, in order to make another product, the sunk entry cost must be incurred once again and another productivity draw taken.[12] The value of a firm with productivity φ is, therefore, the maximum of 0 (if the firm exits) or the stream of future

[11] In our formulation, changes in firm product choice will be driven by changes in the external environment rather than internal productivity. Existing research examines stochastic productivity processes which affect the probability of firm death within single product models (Hopenhayn 1992 and Jovanovic 1982). Adding stochastic productivity would substantially complicate the analysis without changing our key results concerning the links between industry entry/exit and firm product choice.

[12] Abstracting from multiple-product firms allows us to focus on firms' decision concerning *which product* rather than the *number of products* to make.

profits from producing one of the two products discounted by the probability of firm death:

$$v(\varphi) = \max\left\{0, \frac{1}{\delta}\pi_1(\varphi), \frac{1}{\delta}\pi_2(\varphi)\right\}. \tag{9}$$

3.3. Product Choice

Firms decide which product to make based on their realized productivity, taking as given aggregate variables such as the price indices. From our earlier expression for equilibrium profits, firms with zero productivity have negative post-entry profits, with the loss greatest for the high fixed cost product 2:

$$0 > \pi_1(0) = -f_1 > \pi_2(0) = -f_2. \tag{10}$$

A *sufficient* condition for both products to be produced is that profits are positive in each product market and exceed those in the other product market over some range of productivity, φ:

$$\begin{aligned} \pi_1(\varphi) &> 0 \quad \text{and} \quad \pi_1(\varphi) > \pi_2(\varphi) \quad \text{for } \varphi \in \Phi_1 \subset (0,\infty) \\ \pi_2(\varphi) &> 0 \quad \text{and} \quad \pi_2(\varphi) > \pi_1(\varphi) \quad \text{for } \varphi \in \Phi_2 \subset (0,\infty) \end{aligned} \tag{11}$$

which, from equation (10), requires profits for product 2 to increase more rapidly with productivity than those for product 1:

$$\frac{d\pi_2/d\varphi}{d\pi_1/d\varphi} = \Gamma \equiv \left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi} > 1. \tag{12}$$

The *relative* rate at which profits increase with productivity is independent of productivity, and depends instead on parameters, such as the demand-shifter a and the variable cost parameter b, and aggregate variables in the form of the relative price indices, $\mathcal{P}$.

Consumers' taste for both products implies that, in equilibrium, both products will be produced.[13] Therefore, relative prices will adjust so as to ensure that equations (12) and (11) are satisfied, with the two profit functions intersecting at a value for productivity where positive profits are made in each product market, as shown graphically in Figure 1. Endogenous relative prices, $\mathcal{P}$, adjust to ensure that product 2 is produced even if it has both a higher fixed and variable cost.[14]

[13] This result is established formally below when we solve for general equilibrium.

[14] A final technical condition for both products to be produced is that the fixed costs of production do not exhaust the economy's entire supply of labor. This condition must be satisfied as $f_1 \to 0$ and $f_2 \to f_1$.

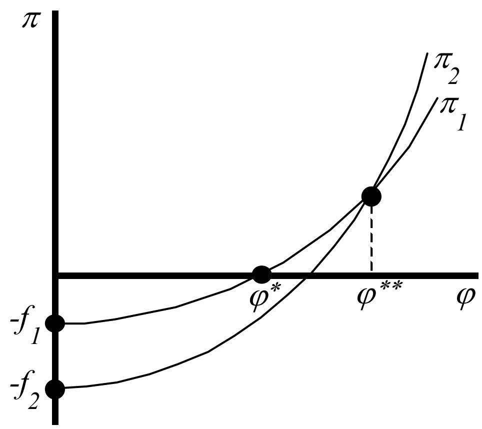

Figure 1: Profit Versus Productivity for the Two Products

Fixed production costs mean that there is a positive value for productivity below which negative profits would be made. Firms drawing a productivity below this **zero-profit productivity cutoff**, φ^*, exit the industry immediately.

The point at which the two profit functions intersect defines another **product-indifference productivity cutoff**, φ^{**}, at which a firm is exactly indifferent between the two products. Product 2 has the greater fixed cost, its profits increase more rapidly with productivity, and the two profit functions intersect where positive profits are made. As a consequence, product 2 will only be produced by high productivity firms, as shown in Figure 1.

The zero-profit productivity cutoff determining the lowest level of productivity where product 1 is produced is given by:

$$r_1\left(\varphi^*\right) = \sigma f_1, \tag{13}$$

while the product-indifference productivity cutoff defining the lowest level of productivity where product 2 is produced is defined by:

$$\frac{r_2\left(\varphi^{**}\right)}{\sigma} - f_2 = \frac{r_1\left(\varphi^{**}\right)}{\sigma} - f_1. \tag{14}$$

Firms drawing a productivity below φ^{**} but above φ^* will make product 1, while those drawing a productivity above φ^{**} will make product 2.

Firms endogenously sort into products based on their heterogeneous characteristics and the diverse attributes of products. Although we have focused on productivity as the relevant firm characteristic and fixed and variable costs as the product attributes, the point that product choice is shaped by this interaction of firm and product heterogeneity is more general. So too is the idea that changes in the external environment will interact with firm and product heterogeneity to influence both industry entry/exit behavior and firm product choice, as developed in Section 5 below.

3.4. Free Entry

From the characterization of entry and product choice in the previous sections, the *ex ante* probability of successful entry into the industry is $[1 - G(\varphi^*)]$, with the *ex ante* probability of producing product 1 given by $[G(\varphi^{**}) - G(\varphi^*)]$, and the *ex ante* probability of producing product 2 given by $[1 - G(\varphi^{**})]$. The *ex post* productivity distribution for each product, $\mu_i(\varphi)$, is conditional on successful entry and firm product choice and is a truncation of the *ex ante* productivity distribution, $g(\varphi)$:

$$\begin{aligned} \mu_1(\varphi) &= \begin{cases} \frac{g(\varphi)}{G(\varphi^{**})-G(\varphi^*)} \text{ if } \varphi \in [\varphi^*, \varphi^{**}) \\ 0 \qquad \text{otherwise} \end{cases}, \\ \mu_2(\varphi) &= \begin{cases} \frac{g(\varphi)}{1-G(\varphi^{**})} \text{ if } \varphi \in [\varphi^{**}, \infty) \\ 0 \qquad \text{otherwise} \end{cases}. \end{aligned} \tag{15}$$

In equilibrium we require the expected value of entry in the industry, v_e, to equal the sunk entry cost, f_e. The expected value of entry is the *ex ante* probability of making product 1 times expected profitability in product 1 until death plus the *ex ante* probability of making product 2 times expected profitability in product 2 until death, and the **free entry condition** is:

$$v_e = \left[\frac{G(\varphi^{**}) - G(\varphi^*)}{\delta}\right]\overline{\pi}_1 + \left[\frac{1 - G(\varphi^{**})}{\delta}\right]\overline{\pi}_2 = f_e, \tag{16}$$

where $\overline{\pi}_i$ is expected or average firm profitability in product market i. Equilibrium revenue and profit in each market are constant elasticity functions of firm productivity (equation (7)) and, therefore, average revenue and profit are equal respectively to the revenue and profit of a firm with weighted average productivity, $\bar{r}_i = r_i(\widetilde{\varphi}_i)$ and $\overline{\pi}_i = \pi_i(\widetilde{\varphi}_i)$, where weighted average productivity, $\tilde{\varphi}_1(\varphi^*, \varphi^{**})$ and $\tilde{\varphi}_2(\varphi^{**})$, is determined by the *ex post* distribution above and is defined formally in the Appendix.

3.5. Product and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, M_e, and a constant mass of firms producing within each product market, M_i. In steady-state equilibrium, the mass of firms who enter and draw a productivity sufficiently high to produce in a product market must equal the mass of firms already within that product market who die, yielding the following **steady-state stability conditions** (**SC**):

$$
\begin{aligned}
[1 - G(\varphi^{**})]M_e &= \delta M_2 && (17)\\
[G(\varphi^{**}) - G(\varphi^{*})]M_e &= \delta M_1. && (18)
\end{aligned}
$$

The firms' equilibrium pricing rule implies that the prices charged for individual varieties are inversely related to firm productivity. The price indices are weighted averages of the prices charged by firms with different productivities, with the weights determined by the *ex post* productivity distributions. Exploiting this property of the price indices, we can write them as functions of the mass of firms producing a product, M_i, and the price charged by a firm with weighted average productivity within each product market, $p_i(\widetilde{\varphi}_i)$:

$$P_1 = M_1^{1/1-\sigma} p_1(\widetilde{\varphi}_1), \qquad P_2 = M_2^{1/1-\sigma} p_2(\widetilde{\varphi}_2) \tag{19}$$

In equilibrium, we also require that the demand for labor used in production, L^p, and entry, L^e, equals the economy's supply of labor, L:

$$L_p + L_e = L. \tag{20}$$

4. General Equilibrium

In this section, we characterize general equilibrium, which is referenced by the sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$. All other endogenous variables may be written as functions of these quantities.

The equilibrium vector is determined by the following equilibrium conditions: the zero-profit productivity cutoff (equation (13)), the product-indifference productivity cutoff (equation (14)), free entry (16), steady-state stability ((17) and (18)), the values for the equilibrium price indices implied by consumer and producer equilibrium (equation (19)), and labor market clearing (20).

We begin by combining the zero-profit productivity cutoff and product-indifference productivity cutoff to derive a supply-side relationship between the two cutoffs and relative prices. A second demand-side relationship between the same variables is derived from consumer and producer equilibrium. We then combine the supply-side and demand-side relationships with the free entry condition to solve for the zero-profit productivity cutoff, φ^*, the product-indifference productivity cutoff, φ^{**}, and relative prices, $\mathcal{P} = P_2/P_1$.

Having determined equilibrium values of these variables, we use the steady-state stability and labor market clearing conditions to solve for each price index individually, P_1 and P_2, and for aggregate revenue in each product market, R_1 and R_2. This completes our characterization of the equilibrium vector. We then show how all other endogenous variables of the model may be determined from the equilibrium vector.

4.1. Relative Supply and Relative Prices

Given the equilibrium pricing rule, the relative revenues of a firm making product 2 with productivity φ^{**} and a firm making product 1 with productivity φ^* are related according to equation (8). The zero-profit productivity cutoff implies that the revenue of a product 1 firm with productivity φ^* is proportional to the product 1 fixed production cost (equation (13)), while the product-indifference productivity cutoff establishes a relationship between relative revenue in the two markets at productivity φ^{**} and the fixed costs of producing the two products (equation (14)).

Combining these three equations, we obtain a downward-sloping (supply-side) relationship between two key variables: the relative value of the two productivity cutoffs, φ^{**}/φ^*, and the relative price of the two products, $\mathcal{P}$.

$$\frac{\varphi^{**}}{\varphi^*} \equiv \Lambda = \left[\frac{\left(\frac{f_2}{f_1} - 1 \right)}{\left[\left(\frac{1-a}{a} \right)^{\psi} \left(\frac{1}{b} \right)^{\sigma-1} \mathcal{P}^{\sigma-\psi} - 1 \right]} \right]^{1/(\sigma-1)} \tag{21}$$

Equation (21) is the mathematical statement of the relationship between the two productivity cutoffs captured graphically in Figure 1. As the relative value of the two cutoffs rises, the fraction of firms producing product 2 falls, and the fraction of firms producing product 1 increases. A higher value for the relative price, $\mathcal{P}$, increases profitability in product 2 relative to product 1 and causes the relative number of firms producing product 2 to rise, i.e. a reduction in the product-indifference productivity cutoff, φ^{**}. For a given value for the relative price, $\mathcal{P}$, a higher fixed cost for product 2 , f_2, reduces profitability in product 2 and increases the product-indifference productivity cutoff, φ^{**}.

For both products to be produced ($\varphi^{**} > \varphi^*$), we require the numerator of the term in parentheses on the right-hand side to exceed the denominator and the denominator to be positive:

$$\left(\frac{f_2}{f_1}-1\right) > \left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}-1\right] > 0 \tag{22}$$

where the fixed cost for product 2 exceeds that for product 1, $f_2 > f_1$.

4.2. Relative Demand and Relative Prices

The expressions for the two price indices yield an equation for relative prices as a function of the relative mass of firms and the relative price charged by a firm with weighted average productivity in each product market (equation (19)). The two steady-state stability conditions yield an equation for the relative mass of firms as a function of the two productivity cutoffs (equations (17) and (18)).

Combining these two equations yields an upward-sloping demand-side relationship between the relative value of the two productivity cutoffs and the relative price of the two products:

$$\Psi\left(\frac{\varphi^{**}}{\varphi^{*}}\right) \equiv \left[\frac{b^{\sigma-1}\int_{\varphi^{*}}^{\varphi^{**}}\varphi^{\sigma-1}g\left(\varphi\right)d\varphi}{\int_{\varphi^{**}}^{\infty}\varphi^{\sigma-1}g\left(\varphi\right)d\varphi}\right] = \mathcal{P}^{\sigma-1}. \tag{23}$$

An increase in the relative consumer price index for product 2, $\mathcal{P}$, reduces demand for product 2 relative to product 1 and shrinks the range of productivities where product 2 is produced relative to the range where product 1 is produced, i.e. an increase in φ^{**}/φ^{*}. For a given value of φ^{**}/φ^{*}, an increase in b, the relative variable cost for product 2, raises the price of product 2 varieties relative to product 1 varieties, i.e. an increase in $\mathcal{P}$.

Combining the downward-sloping supply-side relationship between φ^{**}/φ^{*} and $\mathcal{P}$ in equation (21) with the upward-sloping demand-side relationship in equation (23) yields a unique equilibrium value of (φ^{**}/φ^{*},$\mathcal{P}$), as shown formally in the Appendix. The appendix also shows that, at this equilibrium value for (φ^{**}/φ^{*},$\mathcal{P}$), the condition for both products to be produced in equation (22) is indeed satisfied.

4.3. Free Entry

The zero-profit productivity cutoff, φ^{*}, the relative value for the two productivity cutoffs, φ^{**}/φ^{*}, and the relative price, $\mathcal{P}$, are determined jointly by combining the relative supply and relative demand relationships with the free entry condition.

The free entry condition can be written in a more convenient form using the expression for the zero-profit productivity cutoff, the relationship between the revenues of firms producing varieties in the same market with different productivities, and the supply-side relationship between the two productivity cutoffs derived above. Combining equation (13), $r_i(\varphi'') = (\varphi''/\varphi')^{\sigma-1} r_i(\varphi')$, and equation (21), we can write the free entry condition as:

$$\begin{aligned} v_e = {} & \frac{f_1}{\delta}\int_{\varphi^*}^{\Lambda\varphi^*}\left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] g(\varphi)d\varphi \\ & + \frac{f_1}{\delta}\int_{\Lambda\varphi^*}^{\infty}\left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - \frac{f_2}{f_1}\right] g(\varphi)d\varphi = f_e. \end{aligned} \tag{24}$$

This way of writing the free entry condition clarifies the relationship between the sunk cost of entry and the zero-profit productivity cutoff. An increase in the sunk entry cost, f_e, requires an increase in the expected value of entry, v_e. Since the expected value of entry above is monotonically decreasing in φ^*, this requires a fall in the zero-profit productivity cutoff. Intuitively, the higher sunk cost of entering the industry reduces the mass of entrants, which increases *ex post* profitability, enabling lower productivity firms to cover their fixed production costs and survive in the industry.

Together, equations (24), (23) and (21) determine unique equilibrium values of the three unknowns $(\varphi^*, \varphi^{**}, \mathcal{P})$. These elements of the equilibrium vector are sufficient to determine weighted average productivity, average revenue and average profitability in each product market. Weighted average productivity, $\tilde{\varphi}_i$, depends solely on the two productivity cutoffs. Using the relationships between revenues of firms with different productivities in the same market and in different markets, average revenue and average profitability, $\bar{r}_i = r_i(\tilde{\varphi}_i)$ and $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$, may be written solely as functions of weighted average productivities, relative prices and model parameters.

4.4. Steady-state Stability and Labor Market Clearing

Using the steady-state stability conditions to substitute for the *ex ante* probability of producing each product in the free entry condition, total payments to labor used in entry equal total industry profits: $L_e = M_e f_e = M_1\bar{\pi}_1 + M_2\bar{\pi}_2 = \Pi$ (by choice of numeraire, $w = 1$). The existence of a competitive fringe of potential entrants means that firms enter until the expected value of entry equals the sunk entry cost, and as a result the entire value of industry profits is paid to labor used in entry.

Total payments to labor used in production equal the difference between industry revenue, R, and industry profits, Π: $L_p = R - \Pi$. Taking these two results together, total payments to labor used in both entry and production equal industry revenue, $L = R$, and we have established that the labor market clears (equation (20)).

Consumers allocate their total income to expenditure on the two products according to their CES expenditure shares at the value for equilibrium relative prices determined above: $R_1 = \alpha_1(\mathcal{P})R$ and $R_2 = (1 - \alpha(\mathcal{P}))R$.

The absolute levels of the price indices for the two products individually (P_1, P_2) depend on the mass of firms producing each product and the price charged by a firm with weighted average productivity in each product market. The latter follows immediately from the pricing rule and weighted average productivity for which we have already solved. The mass of firms producing each product is equal to industry revenue divided by average firm revenue, $M_i = (R_i/\overline{r}_i)$, where industry revenue, R_i, and average revenue, $\bar{r}_i = r_i(\tilde{\varphi}_i)$, were also determined above.

Proposition 1 *There exists a unique value of the equilibrium vector* $\{\varphi^* \ ,\varphi^{**} \ ,P_1, \ P_2,$ $R_1, \ R_2\}$. *All other endogenous variables of the model may be written as functions of this equilibrium vector.*

Proof. See Appendix ∎

In the next section we move on to examine how changes in the external environment influence both firms' entry/exit decisions and their product choice. We show how the model captures the key stylized facts developed earlier and we derive additional empirical implications. The final substantive section of the paper presents empirical evidence in support of these additional predictions.

5. Industry Entry and Product Choice

One important insight of the theoretical model is that product choice is determined jointly by heterogeneous firm characteristics, diverse product attributes, and market conditions in the industry as a whole. We now turn to examine other implications of the theory.

When market conditions change, the theory identifies a number of different adjustment margins along which the industry responds. So far we have focused on the typically-overlooked dimension of changes in product mix at surviving firms. However, the theory also incorporates firm entry and exit and, in fact, predicts a systematic relationship between changes in products at surviving firms and changes in entry and exit behavior.

Relevant market conditions in the industry include the sunk costs of entry (f_e), the fixed costs of production (f_1,f_2), the variable cost of production for product 2 relative to that for product 1 (b), and the demand-shifter (a) capturing the relative strength of consumers' preferences for the two products.

Until now the analysis has been consistent with a wide range of distributions for firm productivity, $g(\varphi)$. In this section we focus on the results assuming that the productivity distribution $g(\varphi)$ is *Pareto* with parameters a and k: $g(\varphi) = ak^a\varphi^{-(a+1)}$, where $k > 0$ is the minimum level of productivity so $\varphi \geq k$, a is a shape parameter, and $G(\varphi) = 1 - \left(\frac{k}{\varphi}\right)^a$.[15] This assumption simplifies the analysis, as shown in the Appendix, which also contains formal derivations of the comparative statics in this section.

In the interests of brevity, we provide a complete analysis of one aspect of market structure - the sunk costs of entry (f_e). These may be thought of as capturing barriers to entry in the industry, and may be of particular interest in so far as they can be directly influenced by policy. The impact of changes in other market conditions are analogous. We end the section with a brief discussion of how other parameters of the model influence product choice and industry equilibrium.

Table 4: Comparative Statics for a Change in the Sunk Cost of Entry

$\frac{\partial\varphi^*}{\partial f_e} < 0$	$\frac{\partial\varphi^{**}}{\partial f_e} < 0$
$\frac{\partial(\varphi^{**}/\varphi^*)}{\partial f_e} = 0$	$\frac{\partial\mathcal{P}}{\partial f_e} = 0$
$\frac{\partial\tilde{\varphi}_1(\varphi^*,\varphi^{**})}{\partial f_e} < 0$	$\frac{\partial\tilde{\varphi}_2(\varphi^*,\varphi^{**})}{\partial f_e} < 0$
$\frac{\partial\bar{r}_1}{\partial f_e} = 0$	$\frac{\partial\bar{r}_2}{\partial f_e} = 0$
$\frac{\partial\bar{\pi}_1}{\partial f_e} = 0$	$\frac{\partial\bar{\pi}_2}{\partial f_e} = 0$
$\frac{\partial M_1}{\partial f_e} = 0$	$\frac{\partial M_2}{\partial f_e} = 0$
$\frac{\partial v_e}{\partial f_e} > 0$	$\frac{\partial W}{\partial f_e} < 0$

[15] The distribution of within industry (SIC4) cross-firm labor productivity in the US is well-approximated by a *Pareto* distribution (formally a Pareto-1 distribution). Comparative statics without assuming a particular distribution for firm productivity are less tractable, but are available upon request from the authors.

As shown in Table 4, an increase in the sunk costs of entry in the industry (f_e) lowers both productivity cutoff levels, thus decreasing average productivity in each product and for the industry as a whole. The ratio of the productivity cutoffs, the relative price of the products, the mass of firms producing each product, and average profitability are unchanged. The expected value of entry rises and welfare unambiguously falls.

To understand the intuition behind these results consider what happens when the sunk costs of entry increase. As the sunk costs of entry rise above the expected value of entry, a smaller mass of firms, M_e, will enter the industry. For given values of φ^* and φ^{**}, a smaller mass of entrants implies a smaller mass of firms with productivity realizations high enough to produce in each market. This fall in the mass of firms producing in each market increases *ex post* profitability.

The increase in *ex post* profitability means that firms with lower realizations of productivity than before are able to cover the fixed costs of producing product 1. Hence, in equilibrium the zero-profit productivity cutoff φ^* falls. As φ^* falls for a given value of φ^{**}, this increases the mass of firms in product 1 relative to the mass of firms in product 2, thereby reducing product 1's relative profitability. Hence, some previously high productivity manufacturers of product 1 now find it more profitable to produce the high fixed cost product 2 and φ^{**} also falls.

The equilibrium ratio of the two productivity cutoffs, φ^{**}/φ^*, is independent of the sunk costs of entry, and hence φ^{**} falls by the same proportion as φ^*. With a *Pareto* productivity distribution, this leaves the relative price of the two products, $\mathcal{P}$, unchanged.

The fall in both φ^* and φ^{**} means that some low productivity firms who previously exited now produce product 1, while some previously high productivity manufacturers of product 1 now produce product 2. For both reasons, weighted average productivity in product 1, $\widetilde{\varphi}_1$, will fall. Similarly, the fall in φ^{**} means that product 2 now includes some lower productivity firms who previously manufactured product 1. Hence, weighted average productivity in product 2, $\widetilde{\varphi}_2$, will also fall.

The fall in φ^* and φ^{**} increases the mass of firms with productivity realizations high enough to produce in each market for a given mass of firms, M_e, that enter. With a *Pareto* distribution, this effect exactly offsets the smaller mass of firms entering the industry, so that the mass of firms producing in each product market (M_1, M_2), average firm size $(\bar{r}_1, \bar{r}_2)$, and average *ex post* profitability $(\overline{\pi}_1,\overline{\pi}_2)$ are unchanged.

The expected value of entry, v_e, rises to equal the new higher sunk costs of entry, f_e, because the fall in φ^* and φ^{**} increases the probability of a firm having a productivity realization high enough to be able to profitably manufacture either product 1 or product 2. Welfare per worker, W, falls because, although the mass of firms and hence product varieties is unchanged, the fall in average productivity within each product market leads to a rise in average prices.

The change in market conditions has implications for firm-level outcomes. The move in the productivity cutoffs following a change in market conditions means firms with a range of productivities, $\varphi' \in (G(\varphi^{**}_{\text{old}}) - G(\varphi^{**}_{\text{new}})]$, switch from product 1 to 2. Firm revenue will rise at switching firms from $r_1(\varphi')$ to $r_2(\varphi')$, since a necessary condition for product 2 to be produced is $\Gamma > 1$ in equation (12), which implies $r_2(\varphi') > r_1(\varphi')$ in equation (8).

Firm-level product changes also have implications for aggregate economic outcomes. The rise in the sunk cost of entry leads to a fall in average productivity in each product market and the industry as a whole. More generally, changes in the composition of firms' output across product lines provide a potential source of changes in measured production technique at the industry level.

The model also highlights the systematic relationship between entry/exit decisions and product choice in industry equilibrium. Following the change in market conditions, there is a movement in the zero-profit productivity cutoff which is associated with a change in the rate of entry, $M_e/(M + M_e)$, and exit, $(\delta M + G(\varphi^*)M_e)/(M + M_e)$. There is also a movement in the product-indifference productivity cutoff which implies firm product changes, with the mass of firms in the range of productivities where product changes occur equal to $(G(\varphi^{**}_{\text{old}}) - G(\varphi^{**}_{\text{new}})]M$.

Finally, we briefly discuss changes in other features of the industry as a whole or the attributes of individual products that affect both entry and exit and product choice decisions. Increases in the fixed production cost for product 2 (f_2) reduce relative profitability in this product market, increasing the relative mass of firms that make product 1, and leading to a rise in φ^{**} relative to φ^*. Increases in the fixed production cost for product 1 (f_1) have exactly the opposite effect, increasing the relative mass of firms that make product 2 and reducing φ^{**} relative to φ^*.

Increases in the variable production cost for product 2 (b) reduce relative profitability for this product, again increasing the relative mass of firms that make product 1, and leading to a rise in φ^{**} relative to φ^*. Increases in the weight of product 1 in consumers' utility (a) raise the relative demand for this product, increasing the relative mass of firms that make product 1, and again leading to a rise in φ^{**} relative to φ^*.

6. Empirical Evidence

In this section we take the model back to the data and briefly consider two empirical implications. First, we ask whether products within industries vary in terms of the characteristics of the firms that make them. Second, we examine the link between firm entry and exit and product adding and dropping.

In the model, firms endogenously sort themselves in terms of which product they make. This sorting leads to differences in average firm productivity across the two products within the industry. In addition to labor productivity, we consider average firm total factor productivity, capital intensity, skill intensity and average wages across products within an industry.

Table 5: Firm Sorting into Products

Plant Characteristic	Pecent of Industries Where Producers Highest and Lowest Ranked Product are Significantly Different
Labor Productivity	73
Index Productivity	65
Capital Intensity	66
Skill Intensity	71
Wages	72

Notes: For each product and year, we compute the mean of the noted producer characteristic across all single-product plants producing the product. Products are then ranked according to these means within industries. Cells display share of industries across 1972 to 1997 where the mean of the highest- and lowest-ranked products within the industry are significantly different according to a t-test at the 10 percent level. Diewert index productivity relates the productivity of each plant in industry i in year t to a reference plant in the first year of the sample (see Aw, Chung and Roberts 2003). Skill intensity is the non-production worker share of plant employment.

Table 5 demonstrates that products within industries can be ordered according to producer characteristics. For each product and year, we compute the mean of the noted producer characteristic across all single-product plants producing the product.[16] Products

[16] We compute these averages across plants rather than firms because plants are the most disaggregate production unit for which productivity and input intensities can be calculated. Because factor usage is reported by plant rather than by plant-product, we use only single-product plants in computing these averages.

are then ranked according to these means within industries and years. Each cell of the table reports the share of industries in the pooled 1972 to 1997 sample where the mean of the highest- and lowest-ranked products within the industry and year are significantly different according to a t-test at the 10 percent level of significance.

The first row of the table, for example, indicates that producers are sorted according to labor productivity in 73 percent of industries. Producers also exhibit similar sorting according to an index measure of total factor productivity (row 2) as well as capital intensity, skill intensity and wages. These sortings suggest that firms producing distinct products differ substantially in terms of production technique.

The theoretical model posits a link between changes in entry and exit rates and the amount of product switching within an industry. To determine whether there is a systematic empirical relationship between product switching and entry/exit rates, we run the following industry-level OLS regression:

$$AddDrop_s^{t:t+5} = \beta \left|\Delta EEER_s^{t:t+5}\right| + \alpha_s + \alpha^t + \varepsilon_s^{t:t+5}. \tag{25}$$

The dependent variable of this regression is the share of surviving firms between years t and $t+5$ that either add or drop products within industry s. The key explanatory variable is the absolute value of the change in equilibrium entry and exit rates ($|\Delta EEER|$) over the years t to $t+5$,

$$\left|\Delta EEER_s^{t:t+5}\right| = \left|EEER_s^{t:t+5} - EEER_s^{t-5:t}\right|. \tag{26}$$

In steady state (with a constant mass of firms) the firm entry and exit rates will be equal and will move together in response to a change in market conditions, e.g. a change in sunk entry costs. In practice, industries are likely to be out of steady state and the entry and exit rates of an industry will differ. Therefore, for the empirically unobservable equilibrium entry and exit rate, we use a proxy that is the minimum of each industry s's observed industry entry and exit rates between years t and $t+5$,[17]

$$EEER_s^{t:t+5} = \min\left\{Entry\ Rate_s^{t:t+5}, Exit\ Rate_s^{t:t+5}\right\}.$$

Use of the minimum of the two rates as a proxy for equilibrium entry and exit controls for industry expansion and decline: growing industries are likely have temporarily high entry

[17]We define the entry rate as the ratio of the number of entering firms between years t and $t+5$ to the sum of the number of firms in year t plus entering firms. The exit rate is defined as the ratio of the number of firms that exit between t and $t+5$ and the number of firms in year t.

rates relative to exit rates, while the reverse is true of shrinking industries. The minimum provides a lower bound on the amount of steady state entry and exit in an industry.

Table 6: Output Share of Added and Dropped Products, 1972 to 1997

Independent Variable	Share of Plants Adding or Dropping Products from Years t to t+5
Absolute Value of Change in Equilibrium Entry and Exit Rate (|ΔEEER|) from Years t to t+5	0.281***
	0.067
Year Fixed Effects	Yes
Industry Fixed Effects	Yes
Observations (Industries)	1,802 (459)
Adjusted R^2	0.74

Notes: Four-digit SIC industry-level OLS regression results of product changing activity on changes in equilibrium entry and exit rates. Dependent variable is the share of surviving firms that both add and drop products within industry s between years t and t+5. The change in equilibrium entry and exit rates is the change in the minimum firm entry and exit rate between years t and t+5 (see text). Regression covers U.S. manufacturing activity between 1972 and 1997. Standard errors are heteroskedastic consistent and adjusted for clustering across industries. ***Significant at the 1 percent level; **significant at the 5 percent level; *significant at the 10 percent level.

Table 6 reports the result of estimating equation 25 on the U.S. manufacturing industry dataset described above. We include a full set of year effects to control for common macroeconomic shocks and industry effects to control for unobserved variation across industries in sunk entry costs. As in the theory, identification comes from the relationship between changes in products and changes in equilibrium entry and exit rates.

As indicated in the Table, changes in our measure of equilibrium entry and exit are positively and significantly associated with product-changing activity. We caution that the correlation in this regression is in no way indicative of causation. Both theoretically and empirically we expect a relationship in both directions. The equation is meant to capture an equilibrium relationship and we interpret this result as suggesting that changes in market conditions that influence the firm's decision to enter or exit an industry are correlated with those that influence product switching by continuing firms.

In this section we have attempted to link two general implications of the theoretical model back to the data on products and firms. We confirm that, within an industry, firms producing distinct products differ substantially in terms of production technique. We also find that, within an industry, periods of increased entry and exit are associated with increases in product switching activity by continuing firms.

7. Conclusions

This paper has presented empirical evidence on the importance of product choice by U.S. manufacturing firms, developed a theoretical model of endogenous product selection by heterogeneous firms, and argued that changes in products provide an adjustment margin through which industries respond to evolving market conditions.

More than three-fifths of surviving manufacturing firms add or drop products within industries where they currently produce every five years. Most firms that adjust their product mix both add and drop products. These product switches are major changes to the output mix. The products that are added and dropped account for more than one third of firm output.

Motivated by these stylized facts, the paper develops a theoretical model that integrates endogenous product choice into an analysis of industry equilibrium with entry and exit and heterogeneous firms. Product choice is shaped by the interaction of firm heterogeneity, product diversity, and market conditions in the industry as a whole. Changes in industry market conditions result in simultaneous adjustment along the margins of both firm entry/exit and product choice.

We present empirical evidence in support of these implications of the theoretical model. Firms that produce distinct products have very different productivity levels, consistent with sorting across products by heterogeneous firms. Industries with more product turnover at surviving firms are also characterized by greater changes in entry and exit rates.

The pervasiveness and quantitative importance of product change provides a challenge to the standard way many economists model firms. The average firm is not a highly specialized entity producing a single unchanged product from birth to death, but an organization that responds to market conditions and changes its product mix.

This paper merely begins the examination of the role of product choice in firm and industry behavior. There are a number of interesting areas for further research, both empirical and theoretical. From the data, we would like to know more about the characteristics of firms that switch and the consequences of product changes for firm investment, labor demand, and productivity. In addition, further empirical work can tell us whether switching varies across industries in ways that are similar to entry and exit patterns and how product switching differs across countries. Interesting theoretical extensions would incorporate additional forms of firm and product heterogeneity, the coexistence of single- and multiple-product firms, and firm productivity that evolves over time at surviving firms.

References

Amihud, Y and Baruch, L. (1981) "Risk Reduction as a Managerial Motive for Conglomerate Mergers." *Bell Journal of Economics*, 12, 605-17.

Aw, Bee Yan, Sukyun Chung, and Mark J. Roberts. (2003). "Productivity, Output, and Failure: A Comparison of Taiwanese and Korean Manufacturers ". *Economic Journal* vol. 113, no.491, 485-510

Bartelsman, Eric and Mark Doms, (2000) "Understanding Productivity: Lessons from Longitudinal Microdata." *Journal of Economic Literature*, XXXVIII, 569-94.

Bernard, Andrew B. and J. Bradford Jensen, (1995) "Exporters, Jobs, and Wages in US Manufacturing: 1976-87." *Brookings Papers on Economic Activity: Microeconomics*, 67-112.

Bernard, Andrew B., Jonathan Eaton, J. Bradford Jensen, and Samuel S. Kortum. 2003. Plants and Productivity in International Trade. *American Economic Review*, Vol. 93, No. 4, 1268-1290.

Bolton, P and Farrell, J. (1990) "Decentralization, Duplication and Delay." *Journal of Political Economy*, 98, 803-26.

Chamberlin, Edward H. (1951), "Monopolistic Competition Revisited." *Economica* 18: 343-362.

Chandler, A. (1990) *Scale and Scope*, Cambridge, MA: Belknap Press.

Davis, Steven J and John Haltiwanger. (1991) "Wage Dispersion between and within U.S. Manufacturing Plants, 1963-86." *Brookings Papers on Economic Activity*, Microeconomics, pp. 115-80.

Dixit, Avinash K. and Joseph E. Stiglitz. (1977) "Monopolistic Competition and Optimum Product Diversity." *American Economic Review*, vol. 67, no. 3, 297-308.

Dunne, Timothy, Mark J. Roberts, and Larry Samuelson. (1988) "Patterns of firm entry and exit in U.S. manufacturing industries." *Rand Journal of Economics*, vol. 19 No.4 Winter, 495-515.

Dunne, Timothy, Mark J. Roberts, and Larry Samuelson. (1989) "The Growth and Failure of U.S. Manufacturing Plants." *Quarterly Journal of Economics*, Vol. 104, No. 4 , pp. 671-698.

Gollop, Frank M. and James L. Monahan. (1991) "A Generalized Index of Diversification: Trends in U.S. Manufacturing." *Review of Economics and Statistics*, 73 , 318-30.

Helfat, Constance E. and Ruth S. Raubitschek. (2000) "Product Sequencing: Co-evolution of Knowledge, Capabilities and Products." *Strategic Management Journal*, Vol 21: 961-979.

Hopenhayn, Hugo. (1992) "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica*, 60(5), 1127-1150.

Hotelling, H. (1929) "Stability in Competition." *Economic Journal*, 39: 41-57.

Jovanovic, Boyan. (1982) "Selection and the Evolution of Industry." *Econometrica*, vol. 50, no. 3, May, 649-70.

Lancaster, Kelvin. (1966), "A New Approach to Consumer Theory." *Journal of Political Economy*, 74: 132-157.

Melitz, Marc J. (2003). The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity. *Econometrica*, Vol. 71, November, 1695-1725.

Milgrom, P and Roberts, J. (1990) "The Economics of Modern Manufacturing: Technology, Strategy and Organization." *American Economic Review*, 80, 511-28.

Montgomery, C.A. (1994) "Corporate Diversification." *Journal of Economic Perspectives*, Vol. 8: 1624-1638.

Olley, Steven G. and Ariel Pakes, (1996) "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica*, 64(6), 1263-97.

Pavcnik, N (2002) 'Trade Liberalization, Exit, and Productivity Improvement: Evidence from Chilean Plants', *Review of Economic Studies*, 69(1), 245-76.

Shaked, Avner and Sutton, John. (1982) "Imperfect Information, Perceived Quality, and the Formation of Professional Groups." *Journal of Economic Theory*, vol. 27, no. 1, 170-81

Shaked, Avner and Sutton, John. (1987) "Product Differentiation and Industrial Structure." *Journal of Industrial Economics*, 36, 131-46.

Spence, A. Michael. (1976) "Product Differentiation and Welfare." *American Economic Review*, 66:407-414.

Streitwieser, Mary L. (1992) "The Extent and Nature of Establishment-Level Diversification in Sixteen U.S. Manufacturing Industries." *Journal of Law and Economics*, vol. 34, no. 2, Part 2, 503-34 .

Sutton, John. (1998) *Technology and Market Structure: Theory and History*, MIT Press, Cambridge, Massachusetts.

Sutton, John (2002) "Rich Trades, Scarce Capabilities: Industrial Development Revisited", *Economic and Social Review,* vol. 33, no. 1, 1-22.

U.S. Census Bureau. (1996) "1992 Census of Manufactures: Numerical List of Manufactured and Mineral Products." U.S. Government Printing Office, Washington DC. http://www.census.gov/prod/2/manmin/mc92-r-1.pdf

A Appendix: Theoretical Derivations

A1. Weighted Average Productivity and Average Profitability

$$\begin{aligned}
\widetilde{\varphi}_1(\varphi^*, \varphi^{**}) &= \left[\frac{1}{G(\varphi^{**}) - G(\varphi^*)}\int_{\varphi^*}^{\varphi^{**}} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)} \\
\widetilde{\varphi}_2(\varphi^{**}) &= \left[\frac{1}{1 - G(\varphi^{**})}\int_{\varphi^{**}}^{\infty} \varphi^{\sigma-1} g(\varphi)\, d\varphi\right]^{1/(\sigma-1)}
\end{aligned} \tag{27}$$

Using the relationship between the revenues of firms producing varieties in the same and in different markets, as well as the expression for the zero-profit productivity cutoff and the CES expenditure share, average profit in the two product markets, $\bar{\pi}_i = \pi_i(\tilde{\varphi}_i)$ may be written as follows:

$$\overline{\pi}_1(\varphi^*, \varphi^{**}) = \left[\left(\frac{\widetilde{\varphi}_1(\cdot)}{\varphi^*}\right)^{\sigma-1} - 1\right] f_1 \tag{28}$$

$$\overline{\pi}_2(\varphi^*, \varphi^{**}, \mathcal{P}) = \left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\frac{\widetilde{\varphi}_2(\cdot)}{\varphi^*}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi} - \frac{f_2}{f_1}\right] f_1 \tag{29}$$

A2. Proof of Proposition 1

Proof. We begin by determining the equilibrium sextuple: $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$. First, we use the relative supply and relative demand relationships in equations (21) and (23) to establish that there exist unique equilibrium values of φ^{**}/φ^* and $\mathcal{P}$. Rearranging the product supply relationship, we obtain:

$$\mathcal{P} = b^{\frac{\sigma-1}{\sigma-\psi}}\left(\frac{a}{1-a}\right)^{\frac{\psi}{\sigma-\psi}}\left[\left(\frac{\varphi^{**}}{\varphi^*}\right)^{1-\sigma}\left(\frac{f_2}{f_1} - 1\right) + 1\right]^{\frac{1}{\sigma-\psi}}. \tag{30}$$

Since $\sigma > 1$, the right-hand side is monotonically decreasing in φ^{**}/φ^* and is graphed in $(\mathcal{P}, \varphi^{**}/\varphi^*)$ space in Figure 2. $\mathcal{P}$ takes the value $(f_2/f_1)^{1/(\sigma-\psi)}(a/(1-a))^{\psi/(\sigma-\psi)}b^{(\sigma-1)/(\sigma-\psi)} > 0$ at $\varphi^{**}/\varphi^* = 1$ and converges to a lower value of $(a/(1-a))^{\psi/(\sigma-\psi)}b^{(\sigma-1)/(\sigma-\psi)} > 0$ as φ^{**}/φ^* tends to infinity.

Turning now to the product demand relationship (equation (23)), the left-hand side is monotonically increasing in φ^{**}/φ^* and is also graphed in $(\mathcal{P}, \varphi^{**}/\varphi^*)$ space below. As φ^{**}/φ^* approaches 1, $\mathcal{P}$ converges to 0. As φ^{**}/φ^* tends to infinity, $\mathcal{P}$ converges to ∞.

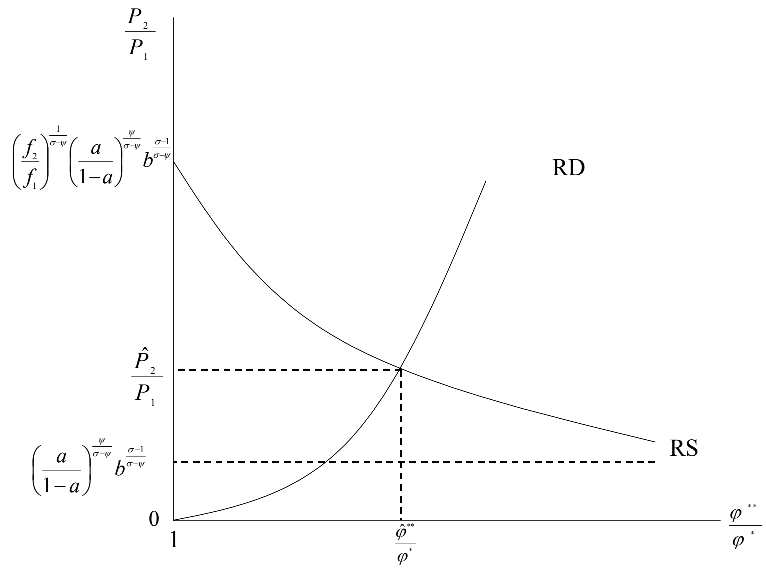

Figure 2: Equilibrium $\mathcal{P}$ and φ^{**}/φ^{*}

There exists a unique equilibrium value of ($\mathcal{P}$, φ^{**}/φ^{*}) where both the relative supply and relative demand relationships are satisfied, and at which $\varphi^{**}/\varphi^{*} > 1$ so the condition for both products to be produced in equation (22) is satisfied.
Given values of $\Lambda \equiv \varphi^{**}/\varphi^{*}$ and $\mathcal{P}$, equation (24) is monotonically decreasing in φ^{*}:

$$\frac{dv_e}{d\varphi^*} < 0 \tag{31}$$

$$\Leftrightarrow \quad \underbrace{\frac{f_1}{\delta}\int_{\varphi^*}^{\Lambda\varphi^*} \varphi^{\sigma-1}(1-\sigma)(\varphi^*)^{-\sigma} g(\varphi) d\varphi}_{\text{Term A}} + \underbrace{\frac{f_1}{\delta}\Lambda\left[\Lambda^{\sigma-1}-1\right] g(\Lambda\varphi^*)}_{\text{Term B}}$$

$$+ \underbrace{\frac{f_1}{\delta}\int_{\Lambda\varphi^*}^{\infty} \left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1} \mathcal{P}^{\sigma-\psi}\varphi^{\sigma-1}(1-\sigma)(\varphi^*)^{-\sigma} g(\varphi) d\varphi}_{\text{Term C}}$$

$$\underbrace{-\frac{f_1}{\delta}\Lambda\left[\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}\Lambda^{\sigma-1} - \frac{f_2}{f_1}\right] g(\Lambda\varphi^*)}_{\text{Term D}} < 0$$

The sum of Terms B and D may be written as,

$$\frac{f_1}{\delta}\Lambda g(\Lambda\varphi^*)\left[\left(\frac{f_2}{f_1}-1\right)-\Lambda^{\sigma-1}\left(\left(\frac{1-a}{a}\right)^{\psi}\left(\frac{1}{b}\right)^{\sigma-1}\mathcal{P}^{\sigma-\psi}-1\right)\right].$$

where, from the definition of Λ in equation (21), the term in square parentheses is exactly equal to zero. Since $\sigma > 1$, Terms A and C in equation (31) are negative. Hence, $\frac{dv_e}{d\varphi^*} < 0$ for all φ^*. Furthermore, as $\varphi^* \to 0$ in equation (24), $v_e \to \infty$. As $\varphi^* \to \infty$, $v_e \to 0$. Together, equations (21), (23) and (24) determine unique equilibrium values of the three unknowns $(\varphi^*, \varphi^{**}, \mathcal{P})$.

These three elements of the equilibrium vector are sufficient to determine weighted average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, in equation (27), as well as average revenue and hence average profitability, $\overline{\pi}_1$ and $\overline{\pi}_2$, in equations (28) and (29).

As shown in the main text, the steady-state stability and free entry conditions (equations (17), (18) and (16)) imply that total revenue, R, is equal to total payments to labor used in both entry and production, L.

Revenue in each product market may be determined from the CES expenditure share (equation (4)) at the equilibrium value of relative prices, $\mathcal{P}$, for which we solved above: $R_1 = \alpha_1(\mathcal{P})L$ and $R_2 = (1 - \alpha(\mathcal{P}))L$.

From consumer and producer optimization, the price indices, P_1 and P_2, may be written as functions of the mass of firms, M_1 and M_2, and the price charged by a firm with weighted average productivity, $p_1(\widetilde{\varphi}_1)$ and $p_2(\widetilde{\varphi}_2)$:

$$\begin{aligned} P_1 &= (M_1)^{\frac{1}{1-\sigma}}\, p_1(\widetilde{\varphi}_1) = \left(\frac{\alpha_1(\mathcal{P})L}{\sigma(\overline{\pi}_1 + f_1)}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\widetilde{\varphi}_1} \\ P_2 &= (M_2)^{\frac{1}{1-\sigma}} p_2(\widetilde{\varphi}_2) = \left(\frac{(1-\alpha_1(\mathcal{P}))L}{\sigma(\overline{\pi}_2 + f_2)}\right)^{\frac{1}{1-\sigma}} \frac{1}{\rho\widetilde{\varphi}_2} \end{aligned}$$

where we have used $M_i = R_i/\overline{r}_i$ and $(\overline{\pi}_1,\, \overline{\pi}_2,\, \widetilde{\varphi}_1,\, \widetilde{\varphi}_2)$ were determined above. We have thus characterized the equilibrium sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$.

We now show that all other endogenous variables of the model may be derived from the equilibrium sextuple $\{\varphi^*, \varphi^{**}, P_1, P_2, R_1, R_2\}$.

From equation (19), (M_1, M_2) can be expressed as functions of the price indices (P_1, P_2) and weighted average productivity $(\widetilde{\varphi}_1,\, \widetilde{\varphi}_2)$ which is determined by $(\varphi^*, \varphi^{**})$ alone. From the analysis in the main text, $M_e = \Pi/f_e = [M_1\bar{\pi}_1 + M_2\bar{\pi}_2]/f_e$, where (M_1, M_2) have just been determined and $(\overline{\pi}_1, \overline{\pi}_2)$ can be derived from $(\varphi^*,\, \varphi^{**},\, \mathcal{P})$.

Total payments to labor used in production in product market i equal the difference between revenue, R_i, and total firm profits, Π_i, in that market. Therefore:

$$\begin{aligned} L_{p1} &= R_1 - \Pi_1 = R_1 - (M_1\bar{\pi}_1) \\ L_{p2} &= R_2 - \Pi_2 = R_2 - (M_2\bar{\pi}_2) \end{aligned}$$

where we have used the choice of labor as numeraire, (R_1, R_2) are part of the equilibrium sextuple, (M_1, M_2) were determined above, and $\overline{\pi}_1$ and $\overline{\pi}_2$ are functions of $(\varphi^*, \varphi^{**}, \mathcal{P})$ alone. Payments to labor used in entry are:

$$L_e = M_e f_e$$

where M_e was determined above.
The first-order conditions for consumer optimization imply:

$$C_1 = R\frac{a^\psi P_1^{-\psi}}{\left[a^\psi P_1^{1-\psi} + (1-a)^\psi P_2^{1-\psi}\right]}, \qquad C_2 = R\frac{(1-a)^\psi P_2^{-\psi}}{\left[a^\psi P_1^{1-\psi} + (1-a)^\psi P_2^{1-\psi}\right]}$$

where $R = L$ and (P_1, P_2) are part of the equilibrium sextuple. ■

A3. Industry Entry and Product Choice

A3.1. Pareto Distribution

Suppose that $g(\varphi)$ is *Pareto-1* with parameters a and k: $g(\varphi) = ak^a\varphi^{-(a+1)}$ where $k > 0$, $a > 0$, and $\varphi \geq k$. The cumulative distribution function for productivity becomes $G(\varphi) = 1 - \left(\frac{k}{\varphi}\right)^a$.

For the variance of log firm sales to be finite, we require $a > \sigma - 1$, in which case the term $\varphi^{\sigma-1}g(\varphi)$ also follows a Pareto distribution with parameters $\gamma \equiv a - \sigma + 1$ and k,

$$\begin{aligned} &\varphi^{\sigma-1}g(\varphi) = \xi h(\varphi) \\ \text{where} \qquad &h(\varphi) = \gamma k^\gamma \varphi^{-(\gamma+1)}, \qquad k > 0,\ \gamma > 0,\ \varphi \geq k \\ &H(\varphi) \equiv \int_0^\varphi h(\varphi)d\varphi = \left[1 - \left(\frac{k}{\varphi}\right)^\gamma\right] \\ &\xi \equiv ak^{a-\gamma}/\gamma > 0 \end{aligned}$$

Therefore, from equation (23), the product demand relationship between the productivity cutoffs $(\varphi^*, \varphi^{**})$ and relative prices $(\mathcal{P})$ simplifies:

$$\mathcal{P} = b\left[(\varphi^{**}/\varphi^*)^\gamma - 1\right]^{1/\sigma-1}. \tag{32}$$

Combining this with the product supply relationship from equation (21), the unique equilibrium value of φ^{**}/φ^{*} is implicitly defined by:

$$\left[\left(\frac{\varphi^{**}}{\varphi^{*}}\right)^{\gamma}-1\right]^{\frac{1}{\sigma-1}}=\left[\left(\frac{\varphi^{**}}{\varphi^{*}}\right)^{1-\sigma}\left(\frac{f_2}{f_1}-1\right)+1\right]^{\frac{1}{\sigma-\psi}}\left(\frac{a}{1-a}\right)^{\frac{\psi}{\sigma-\psi}}b^{\frac{\psi-1}{\sigma-\psi}} \tag{33}$$

A3.2. Sunk Cost of Entry (f_e) Comparative Statics

The expected value of entry in (24) is monotonically decreasing in the zero-profit productivity cutoff φ^{*}. Therefore, as the sunk costs of entry rise, the zero-profit productivity cutoff φ^{*} must fall so as to increase the expected value of entry equal to the new higher sunk cost.

Since $\varphi^{**}=\Lambda\varphi^{*}$ and Λ is unchanged following the rise in the sunk cost of entry, φ^{**} will fall by the same proportion as φ^{*}. Hence: $\partial\varphi^{*}/\partial f_e<0$, $\partial\varphi^{**}/\partial f_e<0$ and $\partial(\varphi^{**}/\varphi^{*})/\partial f_e=0$.

With a *Pareto* productivity distribution, the relative price, $\mathcal{P}$, in equation (32) depends solely on φ^{**}/φ^{*} and hence $\partial\mathcal{P}/\partial f_e=0$.

From the definition of weighted average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, in equations (27) and using the fact that $\varphi^{\sigma-1}g(\varphi)=\xi h(\varphi)$ also follows a Pareto distribution:

$$\widetilde{\varphi}_1(\varphi^{*},\varphi^{**})^{\sigma-1}=\frac{H(\varphi^{**})-H(\varphi^{*})}{G(\varphi^{**})-G(\varphi^{*})}=\frac{a(\varphi^{*})^{\sigma-1}}{\gamma}\left[\frac{1-\Lambda^{-\gamma}}{1-\Lambda^{-a}}\right]. \tag{34}$$

$$\widetilde{\varphi}_2(\varphi^{**})^{\sigma-1}=\frac{1-H(\varphi^{**})}{1-G(\varphi^{**})}=\frac{a(\varphi^{**})^{\sigma-1}}{\gamma}. \tag{35}$$

Since $\sigma>1$, $d\widetilde{\varphi}_1/df_e=(d\widetilde{\varphi}_1/d\varphi^{*})(d\varphi^{*}/df_e)<0$ and
$d\widetilde{\varphi}_2/df_e=(d\widetilde{\varphi}_2/d\varphi^{**})(d\varphi^{**}/df_e)<0$.

The change in average revenue, $\bar{r}_i$, and average profitability, $\bar{\pi}_i$, in equations (28) and (29) depends upon the change in the ratios of weighted average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, to the zero-profit productivity cutoff, φ^{*}:

$$\left(\frac{\widetilde{\varphi}_1(\varphi^{*},\varphi^{**})}{\varphi^{*}}\right)^{\sigma-1}=\frac{H(\varphi^{**})-H(\varphi^{*})}{(\varphi^{*})^{\sigma-1}\left[G(\varphi^{**})-G(\varphi^{*})\right]}=\frac{a\left[1-\Lambda^{-\gamma}\right]}{\gamma\left[1-\Lambda^{-a}\right]}. \tag{36}$$

$$\left(\frac{\widetilde{\varphi}_2(\varphi^{*},\varphi^{**})}{\varphi^{*}}\right)^{\sigma-1}=\frac{1-H(\varphi^{**})}{(\varphi^{*})^{\sigma-1}\left[1-G(\varphi^{**})\right]}=\frac{a\Lambda^{\sigma-1}}{\gamma}. \tag{37}$$

With Λ unchanged, $\partial\overline{\pi}_1/\partial f_e=0$ and $\partial\overline{\pi}_2/\partial f_e=0$.

The mass of firms producing each product is:

$$\begin{aligned} M_1 &= \frac{R_1}{\overline{r}_1} = \frac{\alpha_1(\mathcal{P})L}{\sigma(\overline{\pi}_1 + f_1)} \\ M_2 &= \frac{R_2}{\overline{r}_2} = \frac{(1-\alpha_1(\mathcal{P}))L}{\sigma(\overline{\pi}_2 + f_2)}. \end{aligned} \tag{38}$$

Since both relative prices, $\mathcal{P}$, and relative profitability, $\overline{\pi}_1$ and $\overline{\pi}_2$, are unchanged: $\partial M_1/\partial f_e = 0$ and $\partial M_2/\partial f_e = 0$. Welfare per worker is:

$$\begin{aligned} W &= \left[a^{\psi}P_1^{1-\psi} + (1-a)^{\psi}P_2^{1-\psi}\right]^{\frac{1}{\psi-1}} \\ P_1 &= \left(\frac{R_1}{\overline{r}_1}\right)^{\frac{1}{1-\sigma}} p_1(\widetilde{\varphi}_1) = \frac{\alpha_1(\mathcal{P})L}{\sigma(\overline{\pi}_1 + f_1)}\frac{1}{\rho\widetilde{\varphi}} \\ P_2 &= \left(\frac{R_2}{\overline{r}_2}\right)^{\frac{1}{1-\sigma}} p_2(\widetilde{\varphi}_2) = \frac{(1-\alpha_1(\mathcal{P}))L}{\sigma(\overline{\pi}_2 + f_2)}\frac{b}{\rho\widetilde{\varphi}_2}. \end{aligned} \tag{39}$$

Average profitability in each market, $\overline{\pi}_1$ and $\overline{\pi}_2$, is unchanged, while average productivity, $\widetilde{\varphi}_1$ and $\widetilde{\varphi}_2$, has fallen. Hence, the rise in the sunk cost of entry leads to an increase in the price indices, P_1 and P_2, and a fall in welfare per worker, W.